Ph. D. Section

MATHEMATICAL MODELING OF SOME ASPECTS OF DIGRAPHS WITH IMPETUS OF ISOMORPHISM, DECYCLIZATION AND PEBBLING

A SYNOPSIS SUBMITTED IN PARTIAL FULFILLMENT FOR THE AWARD OF THE DEGREE OF

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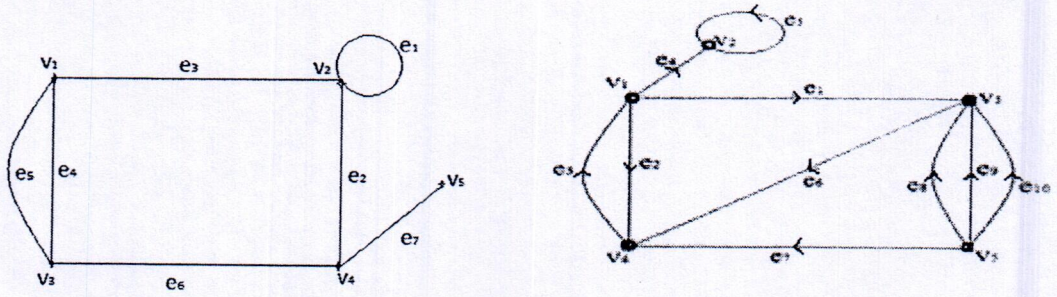
1. Introduction

The general process of transforming a realistic problem into an abstract diagram and then analyzing its behaviour is known as graph theory. Graph theory is extensively used in solution of differrent types of realistic problems arising in applied mathematics and computer oriented inter disciplinery domains. Graph theory is one of the most important branch of discrete mathematics. In discrete mathematics, we study about discrete structures usually finite collections and their properties includes combinatorircs that is the study of combination and enumeration of objects, algorithms for computing properties of collection of objects, where as in graph theory we study about the objects and their relations. Graph theory has many practical applications in various disciplines such as biology, chemitry, physics, computer sciences, economics, enginnering, informatics, linguistics, mathematicis, medicines, social sciences etc.

Graph theory, was born in 1736 with Euler's paper in which he solved the Konigsberg bridge problem. For the next 100 years nothing more was done in the field. In 1847, G. R. Kirchhoff (1824-1887) developed the theory of trees for their applications in electrical networks. Ten year later, A. Caley (1821-1859) discovered trees while he was trying to enumerate the isomers of saturated hydrocarbons C_nH_{2n+2}. A. F. Mobius (1790-1868) first presented the four-color problems in 1840. About 10 years later, A. De Morgan (1806-1871) is the first authenticated reference to the four-color problem. The problem became well known after Cayley published it in 1879 in the first volume of the Proceeding of the Royal Geographic Society. In the year 1859 Sir W. R. Hamiltan (1805-1865) invented a puzzle.

A graph $G = (V, E)$ is a diagram consisting of a collection of vertices together with edges joining certain pairs of these vertices. Mathematically, we can write $G = [V(G), E(G)]$, where, $V(G)$ and $E(G)$ are two finite sets defined as: $V(G)$ = vertex set of graph G and $E(G)$ = edge set of graph G. The order of graph is the number of its vertices, and its size is the number of its edges. A directed graph or digraph consists of a finite set V of vertices and a set A of ordered pairs of distinct vertices called arcs. If the ordered pair $\{u, v\}$ is an arc a, we say that the arc a is directed from u to v.



(a) (b)

Figure 1.1

Figure 1.1(a) is the example of undirected graph and Figure 1.1(b) is the example of digraph. Two graphs G and G' are said to be isomorphic to each other if there is a one-to-one correspondence between their vertices and between their edges such that the incidence relationship is preserved that is two isomorphic graphs must have the same number of vertices, the same number of edges, and an equal number of vertices with a given degree. In Figure 1.2 , $G_1 \cong G_2$ but $G_1 \ncong G_3$ and $G_2 \ncong G_3$.

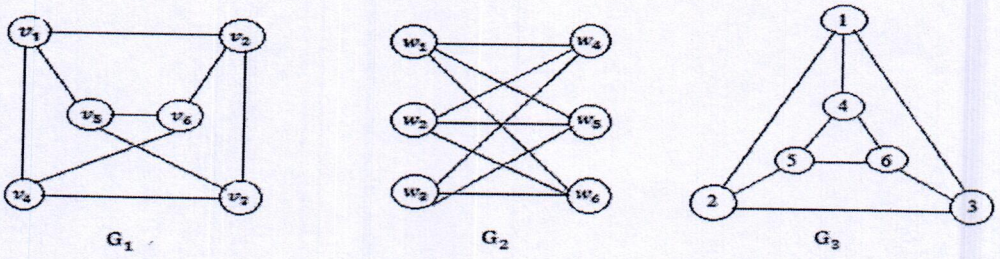


Figure 1.2

A digraph that has no directed circuit is called acyclic. Such a smallest set of edges whose removal destroys all directed circuit in a digraph G is known as a minimum-feedback arc set. In general, a digraph may possess several minimum-feedback are sets. Obtaining one such smallest set of edges may be called minimal decyclization of a digraph. For example, in Figure 1.3, the digraph contains several directed circuits: e_1, e_2, e_3 , e_4, e_6, e_3, and e_4, e_5, e_2, e_3.

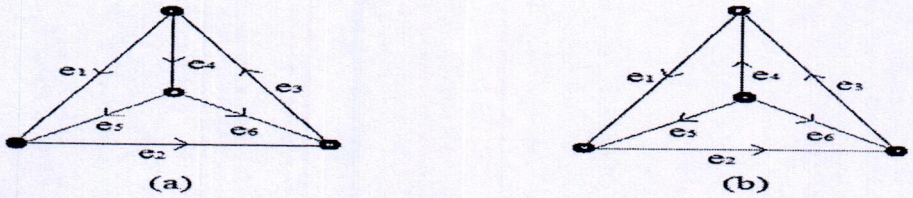


Figure 1.3

The origins of graph pebbling reside in combinatorial number theory and graph theory. Graph pebbling is a mathematical game and area of interest played on a graph with pebbles on the vertices. Game play is composed of a series of pebbling moves. The game of pebbling was first suggested by Lagarias and Saks, as a tool for solving a particular problem in number theory. In 1989 F.R.K. Chung introduced the concept in the literature and defined the pebbling number $\pi(G)$. A graph $G = (V, E)$ is a diagram consisting of a collection of vertices together with edges joining certain pairs of these vertices. Consider a finite connected graph G whose vertices are labeled with non-negatives integers representing the number of pebbles on each vertex. Basically, the pebbling on graph is the mathematical game which is played on the vertices of graphs with number of pebbles (generally, in graph theory pebbles are represented by the whole numbers). A pebbling move is an operation of subtracting 2 from the label of a vertex, and adding 1 to the label of an adjacent vertex which is shown in Figure1.4. A pebbling move can only be made on a vertex with a label of 2 or more.

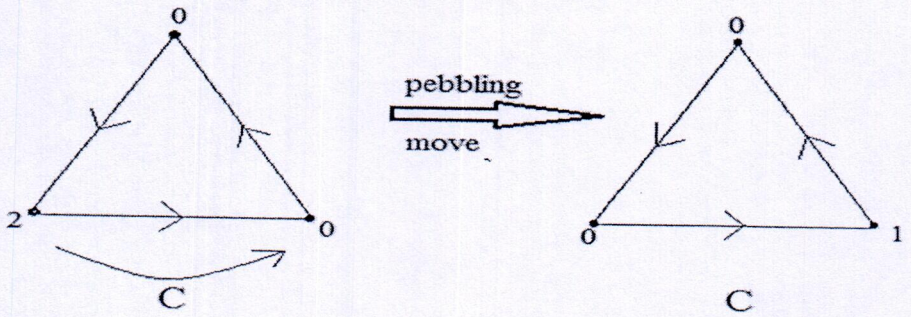


Figure 1.4

2. Objectives

- To cover mathematical modeling of isomorphism of directed graphs using modified graph invariant that is probability propagation matrix with the idea of Markovian property.
- To explore decyclization of directed graph and its isomorphic image using combinatorial algorithms with the theory of Boolean algebra.
- Design and analysis of mathematical modeling of pebbling on directed graphs with incorporation of ideas of graph isomorphism and graph decyclization.

3. Methodology

- For mathematical modeling of isomorphism of directed graphs, we may use and develop modified graph invariant that is probability propagation matrix with the help of statistics and probability theory. For this, we may convert the heuristic algorithm in the form of computer programming in the language of C/C++/MATLAB. In this continuation, we may use the automata theory and Markovian theory.
- For exploration about decyclization of directed graph and its isomorphic image, we may use combinatorial algorithms. It may be shown by mathematical modeling. For this, we may use mathematical and computational tools of Boolean algebra and Linear Algebra.
- For designing and analysis of mathematical modeling of pebbling on directed graphs with incorporation of isomorphism and decyclization, we may use the mathematical and computational methods with number theory, optimization techniques and combinatorics.

4. Importance of Proposed Investigation

Graphs are powerful data structure to represent objects and their concepts. Objects are nothing but nodes and edges describes relation among objects. Number of real world problems is represented by graph. A graph can be used to represent almost any physical situation involving discrete objects and relationship among them. Graph theory is the extension of new possibilities in several fileds. It helps out to explore the reason behind the main problem in easy manner, in a manner of way where everybody can easily understand. It is nothing but a game play of problems in terms of vertices and edges. The reason why these games have been studied is that we can apply analysis of the games

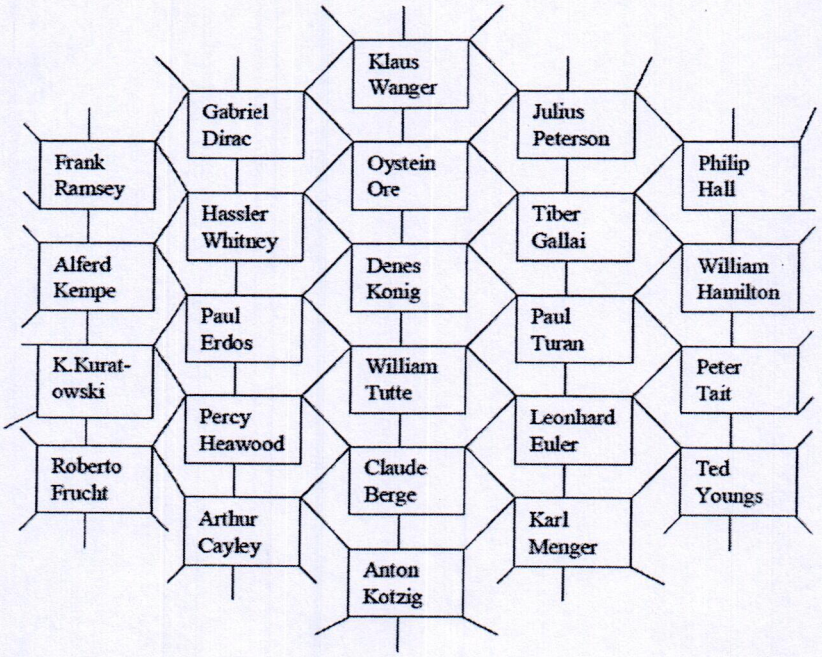
as a way to solve more serious questions in number theory or other mathematical fields. The isomorphic graphs are used to demonstrate the similarity between the any two objects in real life, i.e. the graphs represent any two objects which are similar or differ to eacsh other with the help of numbers of invariant. Graph isomorphism is the area of pattern matching and widely used in various applications such as image processing, protein structure, computer and information system, chemical bond structure, Social Networks. Graph isomorphism is an important problem in complexity theory.

The decyclization can be applied to analysis of Petri nets, evaluation of circuits and programs etc. The known methods are reviewed but they are not effective for big graphs. Acyclic graphs are widely used in applications for example, in logical design algorithms. Decyclization can be applied to analysis of electronic circuits (the minimal set of arcs removing all the circuits of graph is known in electrical engineering as minimal feedback arc set). Feedback is a difference between combinational and sequential logical circuits, and the algorithms of logical design and analysis are very different for those two kinds of circuits; of course the algorithms for combinational circuits are much simpler. Decyclization also can be used for analysis of algorithms represented by graph models, such as PERT (program evaluation and review technique), graph-schemes or Petri nets; for disjunction of feedback in the control systems.

Graph pebbling was developed in 1989 in order to give a more intuitive proof to a result in number theory. In most cases, the reason why these games have been studied is that we can apply analysis of the games as a way to solve more serious questions in number theory or other mathematical fields. Pebbling graphs has the potential to elucidate abstract concepts using more structural ones. Graph pebbling can also be used as a model for the distribution of consumable resources over a network. For example, let pebbles represent barrels of fuel stored in a cargo vessel. Some of the barrels of fuel are used up in transit. Though most research in the field employs the "remove one to move one" approach, a more realistic modeling scheme is an extension called weighted pebbling. Standard pebble game was introduced as a graph-theoretic model that enables to analyze time-space complexity of deterministic computation. Computations are modeled by laying and removing pebbles on/from the vertices. Pebbles represent the memory locations.

5. Review of Work Already Done on the Subject

Dedicated to the memory of the many mathematicians whose contributions, linked in a variety of ways, have led to the development of graph theory[1,2,19].



From Konigsberg to Konig's book,
So runs the graphic tale.
And still it grows more colorful...

.....Blanche Descartes (1969)

There has been research recently in extension of graph theory, especially in graph pebbling, graph isomorphism and graph decyclization [4, 7, 9, 11]. Graphs can represent biological networks at the molecular, protein, or species level. The graph isomorphism problem is to determine if there exists a one-to-one correspondence between the vertices of two graphs G_1 and G_2 that preserves the adjacency of vertices. Two connected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there exists a one-to-one mapping π from V_1 onto V_2 such that $(x, y) \in E_1$ if and only if $(\pi(x), \pi(y)) \in E_2$. The problem of graph isomorphism arises in many fields, such as chemistry, switching theory, information retrieval, and linguistics [7]. Theoretically, it is always possible to determine

whether or not two connected graphs G_1 and G_2 are isomorphic by keeping G_1 fixed and reordering vertices of G_2 to check if their adjacency matrices become identical. This process may require all (n)! reordering and comparisons, 'n' being the number of vertices. Such an inefficient procedure, in which the running time grows factorially with 'n', is of limited use for practical problems [2, 13]. The graph isomorphism problem in terms of probability propagation matrix invariance criterion [7] by using ITCPP [13]. The probability propagation matrix extracts information from degrees and adjacent conditions of vertices simultaneously. After some operations on the matrix, this kind of information about vertices is propagated to their adjacent vertices [2, 23]. Since probability propagation matrices can be computed using elementary operations of vectors and matrices only, therefore this property can be practically used in parallel computers [8, 20]. The graph isomorphism problem is the computational problem of determining whether two finite graphs are isomorphic. The problem is not known to be solvable in polynomial time or to be NP-complete, and therefore may be in the computational complexity class NP-intermediate. It is known that the graph isomorphism problem is in the low hierarchy of class NP, which implies that it is not NP-complete unless the polynomial time hierarchy collapses to its second level. At the same time, isomorphism for many special classes of graphs can be solved in polynomial time, and in practice graph isomorphism can often be solved efficiently. This problem is a special case of the subgraphs isomorphism problem, which asks whether a given graph G contains subgraphs that is isomorphic to another given graph H and which is known to be NP-complete. It is also known to be a special case of the non-abelian hidden subgroup problem over the symmetric group [22].

While for decyclization, there are several algorithm is been discussed some of as[10]:

- Analogy between the Tasks of Decyclization and Finding of Spanning Trees.
- On algorithms for decyclization of oriented graphs.
- DTCPP - A heuristic program for testing decyclization in directed graphs and its isomorphic image by using combinatorial approach.

In extensive search of graph pebbling, many of these extensions involve manipulating the rules of the traditional pebbling game [5, 21]. A number of variations on the theme have been presented over the years. For example, weighted pebbling is played over a graph or

directed graphs whose edges are assigned weights. Christain Belford invented an extension of graph pebbling called rubbling. Rubbling is similar to graph pebbling, but it allows for an additional type of move. Generally, the pebbling concepts is applied to much several graphs (such as weighted graphs, cycle graphs, complete graphs, bipartite graphs, bipartite complete graphs, abstract graph, acyclic graph, Hamiltonian circuit and path, fundamental circuits, isomorphic graphs, chromatic graph, finite graphs, labeled graphs, index graphs, planar graph, nonplanar graph, null graph, connected graph, regular graphs, separable graph, star graph, state graph, stochastic graph, tree graph, middle graph, Petersen graph, all types of directed graphs or digraphs). Discussed the known complexity results in the field of graph pebbling[4,5]. For any configuration of pebbles on the nodes of a graph, a pebbling move replaces two pebbles on one node by one pebble on an adjacent node. A cover pebbling is a move sequence ending with no empty nodes. The number of pebbles needed for a cover pebbling starting with all pebbles on one node is trivial to compute and it was conjectured that the minimum of these simple cover pebbling numbers is indeed the general cover pebbling number of the graph [5, 17]. That is, for any configuration of this size, there exists a cover pebbling. All previously published results about cover pebbling numbers for special graphs (trees etc.) are direct consequences of cover pebbling theorem [3].

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